

مجموعة دواله لقوائم

\sin, \cos, \dots

الاجل -1

[1] $\sin x$

[2] $\cos x$

البات: الجاهر مقلوب
للثاني

[3] $\tan x = \frac{\sin x}{\cos x}$

[4] $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

[5] $\sec x = \frac{1}{\cos x}$

[6] $\csc x = \frac{1}{\sin x}$

$\sin x = \frac{1}{\csc x}$

EXERCISE SET 4.2

 Graphing Utility  CAS

1. In each part, confirm that the formula is correct, and state a corresponding integration formula.

(a) $\frac{d}{dx}[\sqrt{1+x^2}] = \frac{x}{\sqrt{1+x^2}}$

(b) $\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2 \cos(1+x^3)$

2. In each part, confirm that the stated formula is correct by differentiating.

(a) $\int x \sin x \, dx = \sin x - x \cos x + C$

(b) $\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}} + C$

FOCUS ON CONCEPTS

3. What is a *constant of integration*? Why does an answer to an integration problem involve a constant of integration?
4. What is an *integral curve* of a function f ? How are two integral curves of a function f related?

5–8 Find the derivative and state a corresponding integration formula. ■

5. $\frac{d}{dx}[\sqrt{x^3+5}]$

6. $\frac{d}{dx}\left[\frac{x}{x^2+3}\right]$

7. $\frac{d}{dx}[\sin(2\sqrt{x})]$

8. $\frac{d}{dx}[\sin x - x \cos x]$

9–10 Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule (Formula 2 in Table 4.2.1). ■

9. (a) $\int x^8 \, dx$ (b) $\int x^{5/7} \, dx$ (c) $\int x^3 \sqrt{x} \, dx$

(a) $\int \sqrt[3]{x^2} \, dx$ (b) $\int \frac{1}{x^6} \, dx$ (c) $\int x^{-7/8} \, dx$

11–14 Evaluate each integral by applying Theorem 4.2.3 and Formula 2 in Table 4.2.1 appropriately. ■

11. $\int \left[5x + \frac{2}{3x^5}\right] dx$ 12. $\int [x^{-1/2} - 3x^{7/5} + \frac{1}{9}] dx$

13. $\int [x^{-3} - 3x^{1/4} + 8x^2] dx$

$\int \left[\frac{10}{y^{3/4}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}}\right] dy$

15–30 Evaluate the integral and check your answer by differentiating. ■

15. $\int x(1+x^3) dx$ 16. $\int (2+y^2)^2 dy$

17. $\int x^{1/3}(2-x)^2 dx$ $\int (1+x^2)(2-x) dx$

$\int \frac{x^5 + 2x^2 - 1}{x^4} dx$ 20. $\int \frac{1-2t^3}{t^3} dt$

21. $\int [3 \sin x - 2 \sec^2 x] dx$ 22. $\int [\csc^2 t - \sec t \tan t] dt$

23. $\int \sec x(\sec x + \tan x) dx$ $\int \csc x(\sin x + \cot x) dx$

25. $\int \frac{\sec \theta}{\cos \theta} d\theta$ 26. $\int \frac{dy}{\csc y}$

$\int \frac{\sin x}{\cos^2 x} dx$ $\int \left[\phi + \frac{2}{\sin^2 \phi}\right] d\phi$

$\int [1 + \sin^2 \theta \csc \theta] d\theta$ $\int \frac{\sec x + \cos x}{2 \cos x} dx$

Evaluate the integral

$\int \frac{1}{1 + \sin x} dx$

by multiplying the numerator and denominator by an appropriate expression.

22. Use the double-angle formula $\cos 2x = 2 \cos^2 x - 1$ to evaluate the integral

$\int \frac{1}{1 + \cos 2x} dx$

33–36 True-False Determine whether the statement is true or false. Explain your answer. ■

33. If $F(x)$ is an antiderivative of $f(x)$, then

$\int f(x) dx = F(x) + C$

34. If C denotes a constant of integration, the two formulas

$\int \cos x dx = \sin x + C$

$\int \cos x dx = (\sin x + \pi) + C$

are both correct equations.

35. The function $f(x) = \sec x + 1$ is a solution to the initial-value problem

$\frac{dy}{dx} = \sec x \tan x, \quad y(0) = 1$

36. Every integral curve of the slope field

$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$

is the graph of an increasing function of x .

37. Use a graphing utility to generate some representative integral curves of the function $f(x) = 5x^4 - \sec^2 x$ over the interval $(-\pi/2, \pi/2)$.

38. Use a graphing utility to generate some representative integral curves of the function $f(x) = (x^2 - 1)/x^2$ over the interval $(0, 5)$.

39–40 Solve the initial-value problems. ■

39. (a) $\frac{dy}{dx} = \sqrt[3]{x}, \quad y(1) = 2$

(b) $\frac{dy}{dt} = \sin t + 1, \quad y\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(c) $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}, \quad y(1) = 0$

$$(10) \quad a) \int \sqrt[3]{x^2} dx \rightarrow \int x^{\frac{2}{3}+1} dx$$

$$\frac{3}{5} \cdot x^{\frac{5}{3}} + C = \frac{3}{5} \left(\sqrt[3]{x} \right)^5 + C$$

$$b) \int \frac{1}{x^5} dx \rightarrow \int x^{-5+1} dx$$

$$\frac{x^{-5}}{-5} + C \Rightarrow \frac{-1}{5x^5} + C$$

$$c) \int x^{-\frac{7}{8}+1} dx = \frac{1}{-\frac{7}{8}+8} x^{\frac{1}{8}} + C$$

$$\boxed{8 \sqrt[8]{x} + C}$$

$$\boxed{14} \int \left[\frac{10}{y^{3/4}} + \frac{4}{\sqrt{y}} \right] dy$$

$$\int 10 y^{-\frac{3}{4}+1} - y^{\frac{1}{3}+1} + 4 y^{\frac{-1}{2}+1} dy$$

$$\textcircled{10} y^{\frac{1}{4}} - \frac{3}{4} y^{\frac{4}{3}} + 4 y^{\frac{1}{2}} + C$$

$$40 \cdot \sqrt[4]{y} - \frac{3}{4} (\sqrt[3]{y})^4 + 8 \sqrt{y} + C$$

[16] $\int (2+y^2)^2 dy$

$(x+y)^2 =$
 الأول x^2 + y^2 + $2xy$
 الثاني $2xy$ $2xy$
 الأول x^2 y^2 $2xy$
 الثاني $2xy$

$\int 2^2 + (y^{2 \times 2}) + 2(2)(y^2) dy$

$\int 4 + y^{4+1} + 4y^{2+1} dy$

$4y + \frac{y^5}{5} + 4 \frac{y^3}{3} + C$

$$\boxed{17} \int \underline{x^{1/3}} (\underline{2-x})^2 dx$$

$$\int x^{1/3} (2^2 + x^2 - 2(2)(x)) dx$$

$$\int \underline{x^{1/3}} [4 + x^2 - \underline{4x}] dx$$

$$\int 4x^{1/3} + \underbrace{x^{1/3+2}} - 4x^{1/3+1} dx$$

$$\int 4x^{1/3+1} + x^{7/3+1} - 4x^{4/3+1} dx$$

$$\cancel{4} x^{4/3} \cdot \frac{3}{4} + \frac{3}{10} x^{10/3} - 4 x^{7/3} \cdot \frac{3}{7} + C$$

$$\underline{3x^{4/3} + \frac{3}{10}x^{10/3} - \frac{12}{7}x^{7/3} + C}$$

(18) $\int (1 + x^2)(2-x) dx$

(Note: The original image has annotations: a blue circle around '1', a red circle around 'x^2', and a red circle around '2-x'. Blue arrows point from the '1' and 'x^2' circles to the '2-x' circle. A separate note shows a circled '2+1' with 'x' and 'x' written below it.)

$$\int 2 - x^{1+1} + 2x^{2+1} - x^{3+1} dx$$

$$= 2x - \frac{x^2}{2} + 2\frac{x^3}{3} - \frac{x^4}{4} + C$$

(19a) $\int \frac{x^5 + 2x^2 - 1}{x^4} dx$

$$\int \frac{x^5}{x^4} + \frac{2x^2}{x^4 \cdot 2} - \frac{1}{x^4} dx$$

$$\int x + \frac{2}{x^2} - \frac{1}{x^4} dx$$

$$\int x^{1+1} + 2x^{-2+1} - x^{-4+1} dx$$

$$\Rightarrow \frac{x^2}{2} + 2 \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C$$

$$\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C$$

$$\boxed{\text{Zu}} \int \csc x (\sin x + \cot x) dx$$

$$\int \csc x - \sin x + \csc x \cot x dx$$

$$\frac{1}{\cancel{\sin x}} = \cancel{\sin x}$$

$$\rightarrow \int 1 + \csc x \cot x dx$$

$$\Rightarrow \boxed{x - \csc x + C}$$

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$$\int \frac{\sec \theta \cdot d\theta}{\cos \theta = 1}$$

$$\Rightarrow \int \frac{\sec \theta}{1} = \frac{1}{\cos \theta} d\theta$$

$$\int \sec \theta \cdot \sec \theta d\theta$$

$$\int \sec^2 \theta d\theta$$

$$= \tan \theta + C$$

$$(26) \int \frac{dy}{\csc y}$$

$$\Rightarrow \int \sin y \, dy$$

$$= -\cos y + C$$

$$(27) \int \frac{\sin x \cdot 1 \, dx}{\cos x}$$

$$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx$$

$$\int \tan x \cdot \sec x \, dx$$

$$= \sec x + C$$

$$\boxed{28} \int \phi + \frac{2}{\sin^2 \phi} d\phi$$

$$\Rightarrow \int \phi^{1+1} + 2 \csc^2 \phi d\phi$$

$$\Rightarrow \frac{\phi^2}{2} - 2 \cot \phi + C$$

$$\boxed{30} \int \frac{\sec x + \cos x}{2 \cos x} dx$$

$$\int \frac{\sec x}{2 \cos x} + \frac{\cancel{\cos x}}{\cancel{2 \cos x}} dx$$

$$\int \frac{1}{2} \cdot \sec^2 x + \frac{1}{2} dx$$

$$\Rightarrow \frac{1}{2} \tan x + \frac{1}{2} x + C$$

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ص

$$\frac{1}{dx}$$

صبر و استقامت (obstacles)

$$\int \frac{1}{(1 + \sin x)} dx \cdot \frac{(1 - \sin x)}{(1 - \sin x)}$$

$$(1 + \sin x)(1 - \sin x)$$

$$1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x$$

$$= 1 - \sin^2 x = \underline{\underline{\cos^2 x}}$$

$$\int \frac{1 - \sin x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

$$\int \underline{\underline{\sec^2 x}} - \tan x \sec x dx = \boxed{\tan x - \sec x + C}$$

ثابت (جسے، تطابقاً)

$$\text{I)} \quad \underline{\sin^2 x} + \underline{\cos^2 x} = 1 \quad \text{— ثابت}$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\text{II)} \quad \frac{\sin^2 x}{\cos^2 x} + \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 1 \quad / \cos^2 x$$

$\cos^2 x$ سے لیا گیا

$$\Rightarrow \tan^2 x + 1 = \sec^2 x$$

$$\Rightarrow \tan^2 x = \sec^2 x - 1$$

$$\frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}} + \frac{\cos^2 x}{\sin^2 x} = 1 \quad / \sin^2 x$$

$\sin^2 x$ سے لیا گیا

$$\Rightarrow 1 + \cot^2 x = \csc^2 x$$

$$\Rightarrow \cot^2 x = \csc^2 x - 1$$

$$[35] \quad f(x) = \sec x + 1 \quad \leftarrow$$

$$\frac{dy}{dx} = \sec x \tan x, \quad y(0) = 1$$

[Ans]

$$\int dy = \int \sec x \tan x \, dx$$

$$y = f(x) = \sec x + C$$

$$1 = \sec(0) + C$$

$$1 = 1 + C$$

$$1 - 1 = C$$

$$\Rightarrow C = 0$$

$$f(x) = \sec x$$

3a a) $\frac{dy}{dx} = \sqrt[3]{x}$

$y(1) = 2$

$$\int dy = \int x^{\frac{1}{3}} dx$$

$$y = \frac{3}{4} x^{\frac{4}{3}} + C$$

$$y = \frac{3}{4} \sqrt[3]{x^4} + C$$

$$1 = \frac{3}{4} \sqrt[3]{1^4} + C$$

$$1 = \frac{3}{4} + C \Rightarrow C = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore y = \frac{3}{4} \sqrt[3]{x^4} + \frac{1}{4}$$

$$\boxed{61} \int \tan^2 x \, dx$$

$$\int \sec^2 x - 1 \, dx$$

$$= \boxed{\tan x - x + C}$$

$$\boxed{62} \int \cot^2 x \, dx$$

$$\int \underline{\underline{\csc^2 x}} - 1 \, dx$$

$$= \boxed{-\cot x - x + C}$$

سەر ټولگه

[1]

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$$

[2]

$$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$$

[63]

$$\int \sin^2\left(\frac{x}{2}\right) dx$$

$$= \int \frac{1}{2} - \frac{1}{2} \cos(x) dx$$

$$= \frac{1}{2} x - \frac{1}{2} \sin x + C$$

$$\boxed{\text{Ans}} \int \cos^2\left(\frac{x}{2}\right) dx$$

$\frac{x}{2}$ ✓

$$\int \frac{1}{2} + \frac{1}{2} \cos x \, dx$$

$$\frac{1}{2}x + \frac{1}{2} \sin x + C$$

